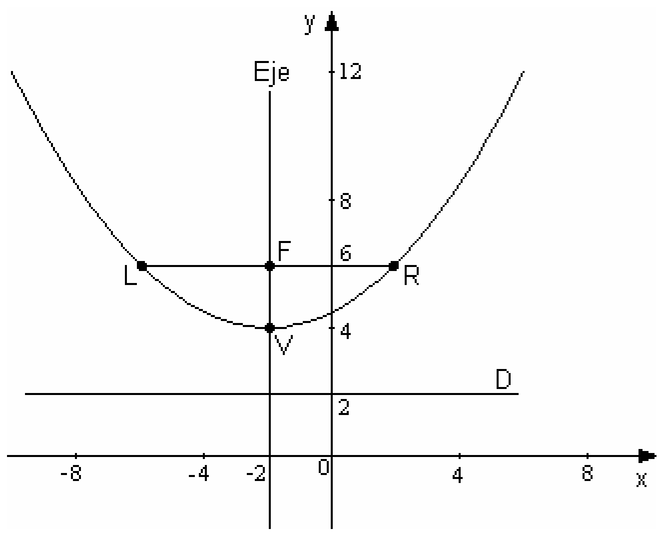
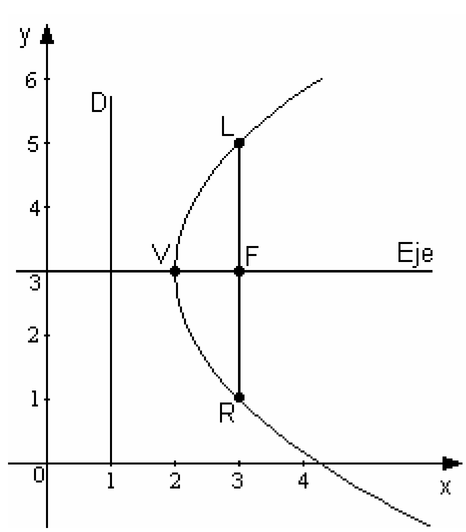
SOLUCIÓN A LOS EJERCICIOS DEL CAPÍTULO IX

9.3. ECUACIÓN DE LA PARÁBOLA EN LAS FORMAS ORDINARIA Y GENERAL CON EJE FOCAL PARALELO CON LOS EJES COORDENADOS

1) $(x+2)^2 = 8(y-4)$; $V(-2,4)$; $p = 2$; $2p = 4$; $F(-2,6)$; $L(-6,6)$; $R(2,6)$; Ec. eje: $x = -2$ Ec. "D": $y = 2$

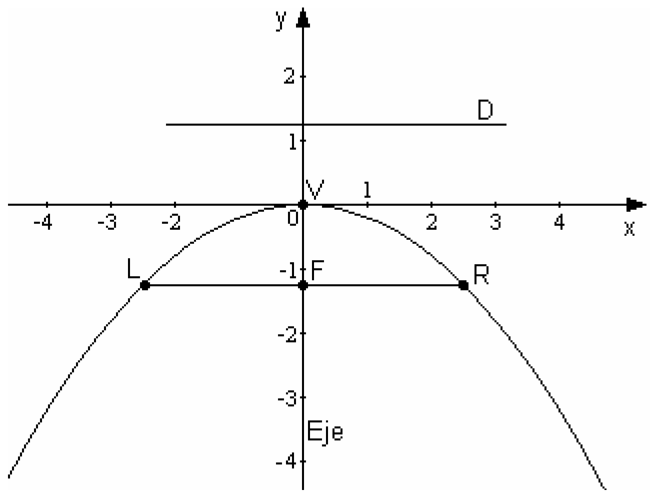


2) $(y-3)^2 = 4(x-2)$; $V(2,3)$; $p = 1$; $2p = 2$; $F(3,3)$; $L(3,5)$; $R(3,1)$; Ec. eje: $y = 3$
Ec. "D": $x = 1$



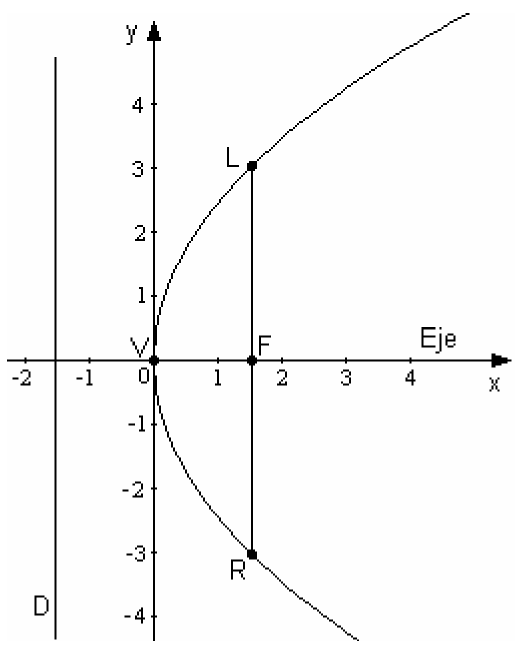
3) $x^2 = -5y$; $V(0,0)$; $p = \frac{5}{4}$; $2p = \frac{5}{2}$; $F\left(0, -\frac{5}{4}\right)$; $L\left(-\frac{5}{2}, -\frac{5}{4}\right)$; $R\left(\frac{5}{2}, -\frac{5}{4}\right)$; Ec. eje: $x = 0$

Ec. "D": $y = \frac{5}{4}$



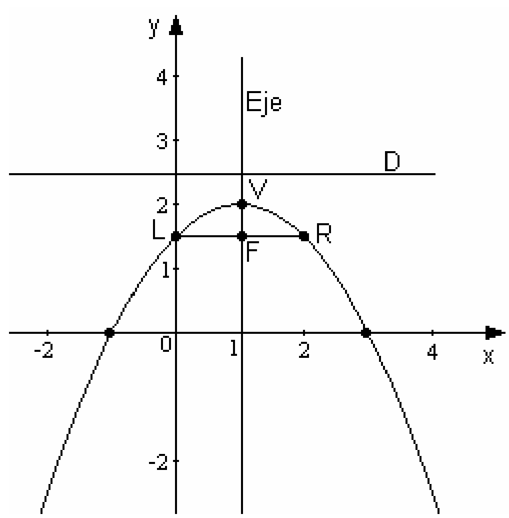
4) $y^2 = 6x$; $V(0,0)$; $p = \frac{3}{2}$; $2p = 3$; $F\left(\frac{3}{2}, 0\right)$; $L\left(\frac{3}{2}, 3\right)$; $R\left(\frac{3}{2}, -3\right)$; Ec. eje: $y = 0$

Ec. "D": $x = -\frac{3}{2}$

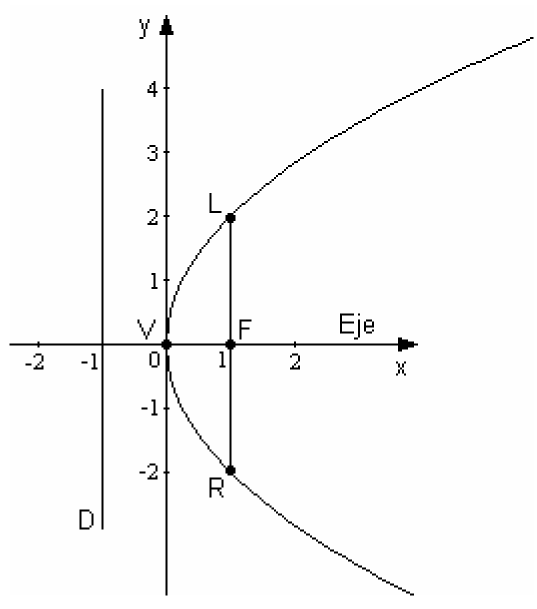


5) $(x-1)^2 = -2(y-2)$; $V(1,2)$; $p=\frac{1}{2}$; $2p=1$; $F\left(1,\frac{3}{2}\right)$; $L\left(0,\frac{3}{2}\right)$; $R\left(2,\frac{3}{2}\right)$; Ec. eje: $x=1$

Ec. "D": $y=\frac{5}{2}$; Intersección con el eje "x": si $y=0$; $x=1\pm 2$; $(-1,0)$; $(3,0)$

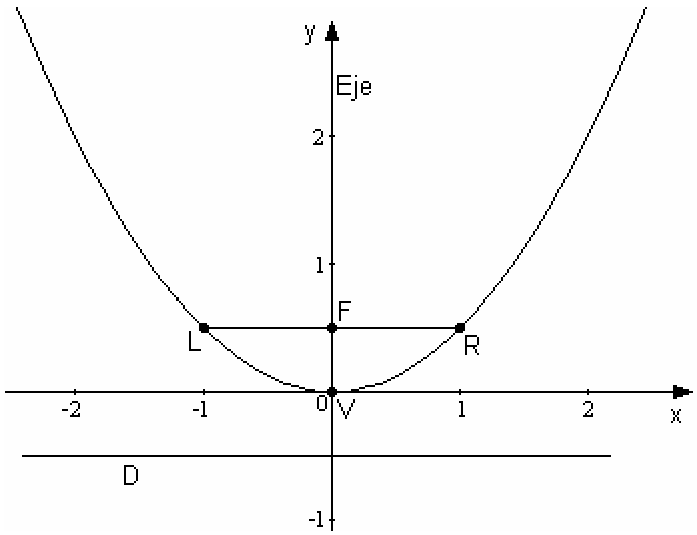


6) $y^2 = 4x$; $V(0,0)$; $p=1$; $2p=2$; $F(1,0)$; $L(1,2)$; $R(1,-2)$; Ec. eje: $y=0$
Ec. "D": $x=-1$

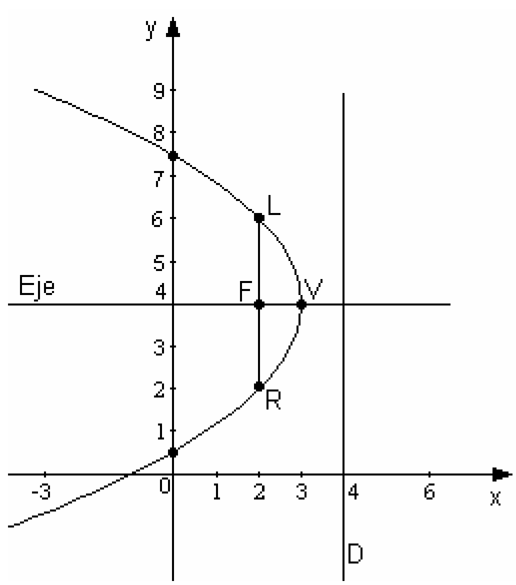


7) $x^2 = 2y$; $V(0,0)$; $p = \frac{1}{2}$; $2p = 1$; $F\left(0, \frac{1}{2}\right)$; $L\left(-1, \frac{1}{2}\right)$; $R\left(1, \frac{1}{2}\right)$; Ec. eje: $x = 0$

Ec. "D": $y = -\frac{1}{2}$

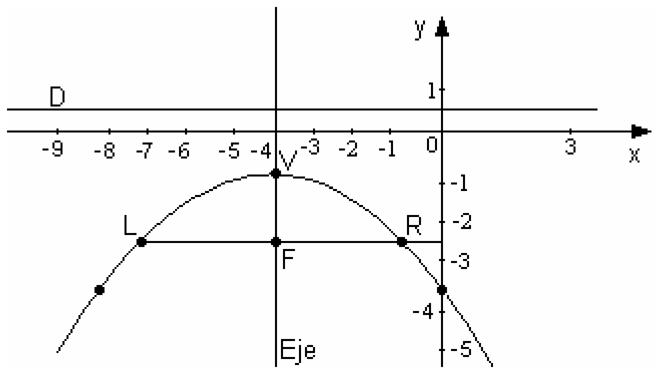


8) $(y-4)^2 = -4(x-3)$; $V(3,4)$; $p = 1$; $2p = 2$; $F(2,4)$; $L(2,6)$; $R(2,2)$; Ec. eje: $y = 4$
Ec. "D": $x = 4$; Intersección con el eje "y": si $x = 0$; $y = 4 \pm 2\sqrt{3}$; $\left(0, 4 + 2\sqrt{3}\right)$ $\left(0, 4 - 2\sqrt{3}\right)$

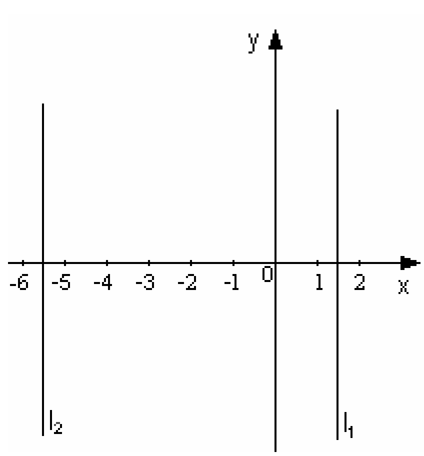


FORMA GENERAL

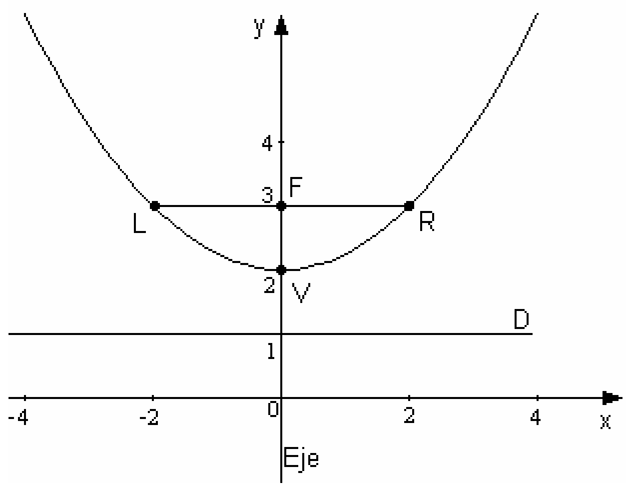
1) $x^2+8x+6y+22=0$; $(x+4)^2=-6(y+1)$; $V(-4,-1)$; $p=\frac{3}{2}$; $2p=3$; $F\left(-4,-\frac{5}{2}\right)$; $L\left(-7,-\frac{5}{2}\right)$; $R\left(-1,-\frac{5}{2}\right)$; Ec. eje: $x=-4$; Ec. "D": $y=\frac{1}{2}$; Intersección con el eje "y": si $x=0$ $\left(0,-\frac{11}{3}\right)$; punto simétrico $\left(-8,-\frac{11}{3}\right)$



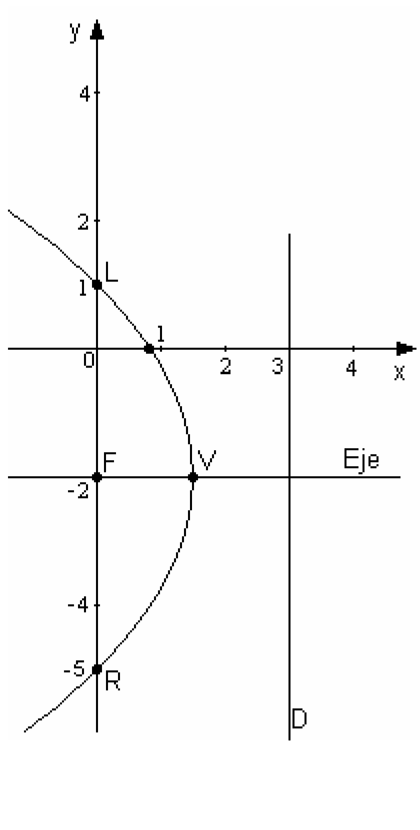
2) $x^2+4x+4=0$; Como $E=0$, la parábola degenera en las 2 rectas paralelas:
$(l_1)\ldots \quad x=-2+2\sqrt{3}$
$(l_2)\ldots \quad x=-2-2\sqrt{3}$



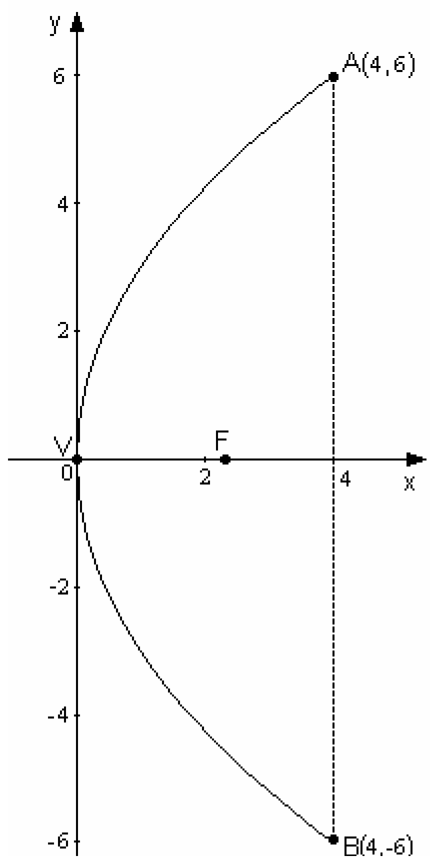
3) $x^2-4y+8=0$; $x^2=4(y-2)$; $V(0,2)$; $p=1$; $2p=2$; $F(0,3)$; $L(-2,3)$; $R(2,3)$
Ec. eje: $x=0$; Ec. "D": $y=1$



4) $y^2+4y+6x-5=0$; $(y+2)^2=-6\left(x-\frac{3}{2}\right)$; $V\left(\frac{3}{2},-2\right)$; $p=\frac{3}{2}$; $2p=3$; $F(0,-2)$ $L(0,1)$ $R(0,-5)$; Ec. eje: $y=-2$; Ec. "D": $x=3$; Intersección con el eje "x": si $x=0$; $\left(\frac{5}{6},0\right)$



5) $y^2 = 9x$; $p = \frac{9}{4}$; $F\left(\frac{9}{4},0\right)$; El foco está a $2.25\ m$ del vértice.



9.4. ECUACIÓN DE LA PARÁBOLA BAJO CIERTAS CONDICIONES, CON EJE PARALELO A UNO DE LOS EJES COORDENADOS

1) $V(-1,-3)$, $F(-2,-3)$; $(y+3)^2 = -4(x+1)$; $y^2 + 4x + 6y + 13 = 0$

2) $V(1,-1)$, $P(3,1)$; $(y+1)^2 = 2(x-1)$; $y^2 - 2x + 2y + 3 = 0$

3) $L(-1,1)$, $R(-1,-5)$; $(y+2)^2 = 6\left(x+\frac{5}{2}\right)$; $y^2 - 6x + 4y - 11 = 0$

$$(y+2)^2 = -6\left(x-\frac{1}{2}\right) \ ; \ y^2 + 6x + 4y + 1 = 0$$

4) $A(2,-1)$, $B(4,0)$ y $C(5,3)$; $\begin{cases} 2D - E + F = -4 \\ 4D \qquad + F = -16 \\ 5D + 3E + F = -25 \end{cases}$; $D = -\frac{27}{5}$; $E = -\frac{6}{5}$; $F = \frac{28}{5}$

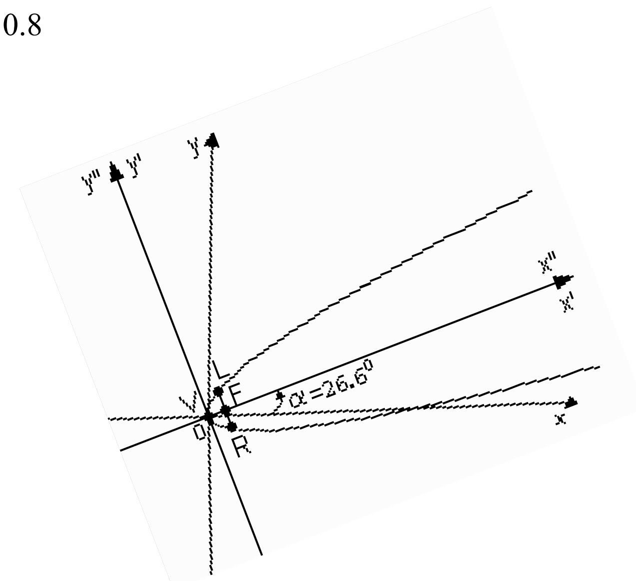
$5x^2 - 27x - 6y + 28 = 0$

5) $A(2,-1)$, $B(4,0)$ y $C(5,3)$; $\begin{vmatrix} x^2 & x & y & 1 \\ x_A^2 & x_A & y_A & 1 \\ x_B^2 & x_B & y_B & 1 \\ x_C^2 & x_C & y_C & 1 \end{vmatrix} = 0$; $5x^2 - 27x - 6y + 28 = 0$

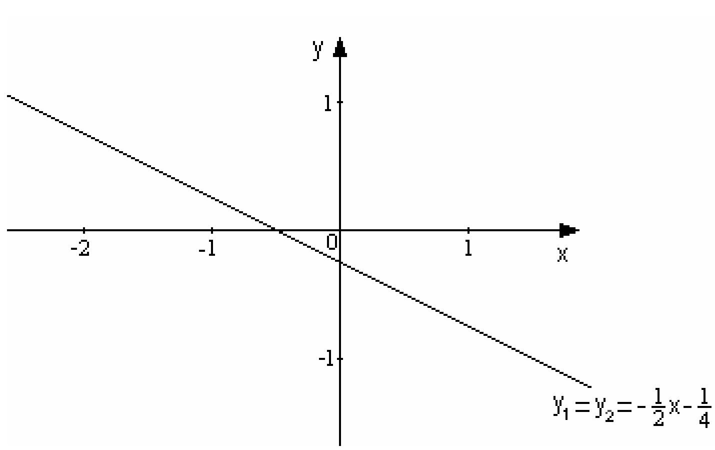
9.5. ECUACIÓN DE UNA PARÁBOLA CON EJE FOCAL OBLICUO RESPECTO A LOS EJES COORDENADOS

1) $x^2 - 4xy + 4y^2 - x = 0$; por el primer procedimiento, $\tan 2\alpha = \dfrac{4}{3}$; $\cos 2\alpha = \dfrac{3}{5}$; $sen\alpha = \dfrac{1}{\sqrt{5}}$; $\cos\alpha = \dfrac{2}{\sqrt{5}}$; $\alpha = 26.6^0$; $x = \dfrac{2x'-y'}{\sqrt{5}}$; $y = \dfrac{x'+2y'}{\sqrt{5}}$; $\left(y' + \dfrac{1}{10\sqrt{5}}\right)^2 = \dfrac{2}{2\sqrt{5}}\left(x' + \dfrac{\sqrt{5}}{200}\right)$; $\boxed{y''^2 = \dfrac{2}{5\sqrt{5}}x''}$

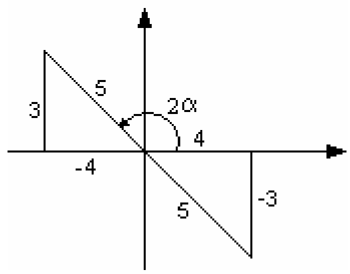
$p = \dfrac{1}{10\sqrt{5}} \approx 0.4$; $2p \approx 0.8$



2) $x^2 + 4xy + 4y^2 + x + 2y + \dfrac{1}{4} = 0$, por el segundo procedimiento, $y = -\dfrac{1}{2}x - \dfrac{1}{4}$. La parábola degenera en 2 rectas coincidentes.

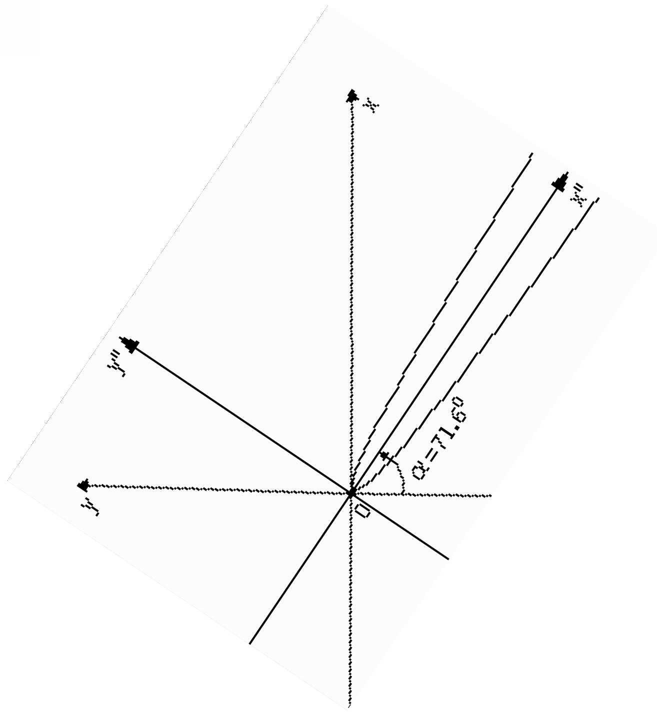


3) $9x^2-6xy+y^2-x=0$, por el primer procedimiento, $\tan 2\alpha=\dfrac{-3}{4}$; para que α sea agudo



$\cos 2\alpha=\dfrac{-4}{5}$; $sen\alpha=\dfrac{3}{\sqrt{10}}$; $\cos\alpha=\dfrac{1}{\sqrt{10}}$; $\alpha=71.6^0$; $x=\dfrac{x'-3y'}{\sqrt{10}}$

$y=\dfrac{3x'+y'}{\sqrt{10}}$; $\left(y'+\dfrac{3}{20\sqrt{10}}\right)^2=\dfrac{1}{10\sqrt{10}}\left(x'+\dfrac{9\sqrt{10}}{400}\right)$; $\boxed{y''^2=\dfrac{1}{10\sqrt{10}}x''}$

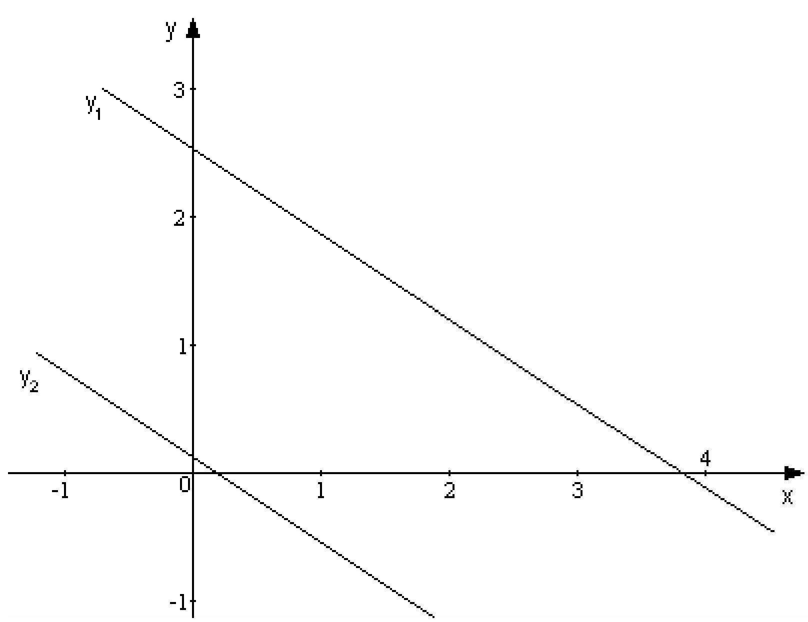


4) $4x^2+12xy+9y^2-16x-24y+3=0$, por el segundo procedimiento,

$$y_1=-\frac{2}{3}x+\frac{4+\sqrt{13}}{3}$$

$$y_2=-\frac{2}{3}x+\frac{4-\sqrt{13}}{3}$$

La parábola degenera en 2 rectas paralelas



5) $9x^2 - 6xy + y^2 + x - 2y - 14 = 0$, por el segundo procedimiento, $y = 3x + 1 \pm \sqrt{5}\sqrt{x+3}$